Business PreCalculus MATH 1643 Section 004, Spring 2014
Lesson 4: Rational Expressions

Definition 1. **Rational Expression:** *The quotient of two polynomials is called a rational expression.*

Example 1. **a-** $\frac{10}{17}$.

b- $\frac{x+2}{(x-3)(x+4)}$.

Definition 2. **Finding Least Common Denominator for Rational Expressions:**

1. *Factor each denominator polynomial completely.*
2. *Form a product of the different irreducible factors of each polynomial. (Each distinct factor is used once)*
3. *Attach to each factor in this product the largest exponent that appears on this factor in any of the factored denominators.*

Example 2. *Find the LCD for the pair:*

$$\frac{x+2}{x(x-1)^2(x+2)} \quad \text{and} \quad \frac{3x+7}{4x^2(x+2)^3}$$

Solution:
Step1. *The denominators are already completely factored.*
Step2. $4x(x-1)(x+2)$. *(Product of the different factors)*
Step3. $LCD = 4x^2(x-1)^2(x+2)^3$, *the largest exponents are* $2, 2,$ *and* 3.

Example 3. *Find the LCD for the the rational expressions:*

$$\frac{x+1}{x^2-x-6} \quad \text{and} \quad \frac{2x-13}{x^2-9}$$

Solution:
Step1. $x^2 - x - 6 = (x-3)(x+2)$
$x^2 - 9 = (x-3)(x+3)$
Step2. $(x+3)(x-3)(x+2)$. *(Product of the different factors)*
Step3. $LCD = (x+3)(x-3)(x+2)$, *the largest exponents are* $1, 1,$ *and* 1.

Definition 3. **Adding/Subtracting Rational Expressions:** *To add or subtract rational expressions:*

1. *Find the LCD.*
2. *Use* $\frac{A}{B} = \frac{AC}{BC}$.
3. *Follow the order of operations.*
4. *Simplify.*

Example 4. *Add the following:*

$$\frac{3}{x^2-1} + \frac{x}{x^2+2x+1}$$

Solution:
Step 1. *The LCD* $= (x+1)^2(x-1)$.

Step 2. $\frac{3}{x^2-1} = \frac{3}{(x-1)(x+1)} = \frac{3(x+1)}{(x-1)(x+1)^2}$

$\frac{x}{x^2+2x+1} = \frac{x}{(x+1)^2} = \frac{x(x-1)}{(x+1)^2(x-1)}$

Step 3.

$$\begin{aligned}\frac{3}{x^2-1} + \frac{x}{x^2+2x+1} &= \frac{3(x+1)}{(x-1)(x+1)^2} + \frac{x(x-1)}{(x+1)^2(x-1)} \\ &= \frac{3(x+1)+x(x-1)}{(x-1)(x+1)^2} \\ &= \frac{3x+3+x^2-x}{(x-1)(x+1)^2} \\ &= \frac{x^2+2x+3}{(x-1)(x+1)^2}, \quad (Step4.\ Simplify)\end{aligned}$$

Definition 4. **Complex Fraction:** *A rational expression that contains another rational expression in its numerator or denominator (or both) is called a complex rational expression.*

Example 5. $\frac{\frac{1}{2}+\frac{1}{x}}{\frac{x^2-4}{2x}}$.

Definition 5. **Simplifying Complex Fractions:** *To simplify a complex fraction perform the operations indicated in both the numerator and denominator of the complex fraction. Then multiply the resulting numerator by the reciprocal of the denominator.*

Example 6. *Simplify the complex fraction:*

$$\frac{\frac{1}{2}+\frac{1}{x}}{\frac{x^2-4}{2x}}.$$

Solution:

$$\begin{aligned}&\frac{\frac{1}{2}+\frac{1}{x}}{\frac{x^2-4}{2x}} \\ &= \frac{\frac{x+2}{2x}}{\frac{x^2-4}{2x}}, \ (Performing\ the\ indicated\ operation) \\ &= \frac{x+2}{2x}.\frac{2x}{x^2-4}, \ (Multiplying\ by\ the\ reciprocal) \\ &= \frac{(x+2)(2x)}{(2x)(x^2-4)} \\ &= \frac{(x+2)(2x)}{(2x)(x-2)(x+2)} = \frac{1}{x-2}\end{aligned}$$